O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2001
Midterm Exam #1

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Problem 1:

Suppose we have a state-space realization given by A, b, c with the three chosen state variables $x = [x_1 \quad x_2 \quad x_3]^T$. Suppose we are now interested in the state variables $z = [z_1 \quad z_2 \quad z_3]^T$, where $z_1 = k_1 x_1$, $z_2 = k_2 x_2$, and $z_3 = k_3 x_3$, and we let $\dot{z} = Fz + gu$, $y = hz$.

a) Write out the matrices F, g, h in terms of the elements of A, b, c and the scale factors k_1, k_2, k_3.

b) Suppose we wish to change the time scale and substitute $\tau = a_0 t$ into the equations. Repeat part a), showing how F, g, h depend on the time scale factor a_0 and the elements of A, b, c.

<u>Problem 2</u>:

If $\{A, b, c, d\}$, $d \neq 0$, is a realization with $H(s) = c(sI - A)^{-1}b + d$, show that $\{A - (bc/d), b/d, -c/d, 1/d\}$ is a realization for a system with transfer function $1/H(s)$.

Problem 3:

Realize the following SIMO continuous-time, time-varying system and show one feasible state space representation, i.e., $\{A(t), B(t), C(t), D(t)\}$,

$$\begin{aligned} e^{-t}\dot{y}_1(t) + y_1(t) + \ddot{y}_2(t) + y_2(t) &= tu(t) \\ \dot{y}_1(t) + \dot{y}_2(t) + ty_2(t) &= \dot{u}(t) + t^2u(t) \end{aligned}.$$

Problem 4:

A nonlinear system is given by

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u) \\ f_2(x_1, x_2, u) \end{bmatrix} = \begin{bmatrix} 1 + 2e^{2x_1} - 3(x_2 - 1)^2 + \sin 5u \\ \frac{1}{3}x_1 x_2^3 - x_1 x_2 + 2\ln(1 + x_1) \end{bmatrix}.$$

Linearize the system about the equilibrium point. To improve the accuracy, approximate up to the second order in the linearization process.

Problem 5:

Let

$$H(s) = \begin{bmatrix} \frac{s^2+1}{s^3} & \frac{2s+1}{s^2} \\ \frac{s+3}{s^2} & \frac{2}{s} \end{bmatrix}$$

be a transfer function matrix. Find a minimal realization (i.e., simulation diagram and state space representation) for the continuous-time system defined above as, $H(s)$.